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Low temperature nonequilibrium dynamics in transverse Ising spin glass

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Abstract. The real part of the time-dependent *ac* susceptibility of the short-range Ising spin glass in a transverse field has been investigated at very low temperatures. We have used the quantum linear response theory and domain coarsening ideas of quantum droplet scaling theory. It is found that after a temperature quench to a temperature T_1 (lower than the spin glass transition temperature T_g) the *ac* susceptibility decreases with time approximately in a logarithmic way as the system tends to the equilibrium. It is shown that the transverse field of "tunneling" has unessential effect on the nonequilibrium dynamical properties of the magnetic droplet system. The role of quantum fluctuations in the behavior of the *ac* susceptibility is discussed.

PACS. 75.40.Gb Dynamic properties (dynamic susceptibility, spin waves, spin diffusion, dynamic scaling, etc.) – 75.10.Nr Spin-glass and other random models – 75.50.Lk Spin glasses and other random magnets

1 Introduction

Aging phenomena and nonequilibrium slow dynamics have been investigated during last years in many materials with glassy properties such as spin glasses, polymer glasses, orientational glasses, simple liquids like glycerol and gels [1–12]. These systems are characterized by the existence of a nonequilibrium low temperature phase and aging [13–15]. Despite a great progress towards the understanding of nonequilibrium dynamics, some problems remain open. One of them is the investigation of the very low temperature nonequilibrium dynamics in quantum spin glasses, namely the nature of the quantum channels of relaxation and the behavior of a quantum glassy system subjected to a periodic driving force, aging at very low temperatures. The natural basis for the interpretation of aging is based on coarsening ideas of a slow domain growth of a spin-glass type ordered phase [6,9,15]. For theoretical studies of quantum fluctuations in disordered media there is a variety of techniques including replica theory, renormalization group, Monte Carlo simulations, the Schwinger and Keldysh closed-time path-integral formalism and others [26–38]. A large attention in the last decade was devoted to the spin glasses representing a model system for the study of nonequilibrium dynamics [39–45].

In this paper we investigate the real-time nonequilibrium dynamics in a d-dimensional Ising spin glass in a transverse field in terms of the droplet model at very low temperatures. We calculate the *ac* susceptibility as a function of the time elapsed after a thermal quench and of the frequency of the driven field. We show that quantum effects insignificantly alter the nonequilibrium dynamics in the spin glass phase at very low temperatures.

In *ac* susceptibility measurements performed on classical spin glasses the magnetic response of the system to a small *ac* magnetic field after quenching shows aging effects. This response depends on its thermal history and on the time interval the system has been kept at a constant temperature in the glass phase.

It is assumed that isothermal aging is a coarsening process of domain walls, and the temporal *ac* susceptibility (real part χ' and imaginary part χ'') at a given frequency of the *ac* magnetic field ω at time t after the quenching scales as [17,24,35]

$$\frac{\chi''(\omega,t)-\chi''_{eq}(\omega)}{\chi''(\omega,t)} \sim \left[\frac{L\,(1/\omega)}{R(t)}\right]^{d-\theta}, \tag{1}$$

$$\frac{\chi'(\omega,t)-\chi'_{eq}(\omega)}{\chi'(\omega,t)} \sim \left[\frac{L\,(1/\omega)}{R(t)}\right]^{d-\theta} \tag{2}$$

for $|\ln\omega| \ll \ln t$ if $L(1/\omega)$ is proportional to $\ln\omega^{-1}$ and $R(t)$ is proportional to $\ln t$. In the above equations $\theta \leq (d-1)/2$, $L\,(1/\omega)$ is the typical size of the droplet being polarized by the *ac* field, and $R(t)$ is the typical domain size. L and R may change according to a logarithmic growth law or to an algebraic one [41,45]. It must be clear that both laws cannot hold simultaneously

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for the same material. The final test is experimental and no decisive evidence in favour of one hypothesis or the other is available yet. The logarithmic growth law (like the algebraic growth law) is supported by recent experiments [17,18]. The ratio $L\,(1/\omega)\,/R(t)$ is proportional to $[\ln{(\omega/\omega_0)}/\ln{(t/t_0)}]^{1/\psi}$ if the droplet picture is used, ψ is some exponent, $0 \le \psi \le d-1$, t_0 is a certain microscopic characteristic unit of time. χ'_{eq} and χ''_{eq} are the real and the imaginary parts of the equilibrium susceptibility. The expressions (1, 2) were found when the relaxation is governed by thermal activation over a free-energy barrier B. The barriers for annihilation and creation of the droplet excitations are assumed to scale as $B \sim \Delta L^{\psi}$; Δ is a barrier energy at $T \ll T_g$. The barriers have a broad energy distribution. A droplet with B lasts for a time t of order of $t_0 \exp{[B/(k_B T)]}$ where k_B is the Boltzmann constant. t is the rate of classical activation over energy barrier B.

After a time t after quenching the typical linear domain size of the system has accordingly grown to $R(t) \sim [(k_B T/\Delta(T)) \ln{(t/t_0)}]^{1/\psi}$. In the *ac* susceptibility measurements at angular frequency ω, the *ac* field excites droplets of length scales up to $L\,(1/\omega) \sim [(k_B T/\Delta(T))\,|\ln{(\omega/\omega_0)}\,|]^{1/\psi}$. Because in aging experiments the elapsed time satisfies in general the relation $t \ge \omega^{-1}$ [12] we have $L\,(1/\omega) < R(t)$. These droplets have walls which partly coincide with walls of the domain of size R. The presence of such frozen-in domain walls influences the small length scale droplet excitations. Some droplets which touch it can reduce their excitation gap, compared with others in the bulk of domains. In the presence of domain walls at a typical distance R from each other Fisher and Huse have found the free energy gap of a droplet of size L in the following form [35]

$$\epsilon_L = \gamma_{eff}\left[\frac{L}{R}\right]\left(\frac{L}{L_0}\right)^{\theta}, \quad L < R. \tag{3}$$

with an effective stiffness constant $\gamma_{eff}\,[L/R] = \gamma\left[1 - c_\nu\,(L/R)^{d-\theta}\right]$, γ being the droplet stiffness constant. L_0 is a certain microscopic unit of length playing the role of a short-distance cutoff; c_ν is a constant independent of time and frequency which happens to be anomalously small. So, within the droplet picture, aging proceeds by coarsening of domain walls as usual phase ordering processes. The domain wall serves as the frozen-in extended defect for the droplets and reduces their stiffness constant from γ to γ_{eff}. It is known that the susceptibility is inversely proportional to γ. It was derived, for example, for the real part of $\chi'(\omega, t)$ that [24]

$$\chi'(\omega, t) = \chi'_{eq}(\omega)\left(1 - \frac{\Delta\gamma}{\gamma}\right)^{-1} \tag{4}$$

where $\Delta\gamma/\gamma \sim (L(1/\omega)/R(t))^{d-\theta}$ with $\Delta\gamma = \gamma_{eff} - \gamma$ [35].

The condition $\beta\Gamma_L \ll 1$, where Γ_L is the droplet tunnelling rate and $\beta = (k_B T)^{-1}$, defines the classical regime, whereas in the quantum regime one has $\beta\Gamma_L \gg 1$ [36,37]. For the quantum droplet model developed in [36–38] $L(1/\omega) \sim [(1/\sigma)\,|\ln{(\Gamma_0/\omega)}|]^{1/d}$ and $L(t) \sim [(1/\sigma)\,|\ln{(\Gamma_0 t)}|]^{1/d}$, where Γ_0 is the microscopic tunnelling rate and the coefficient σ has little variation from droplet to droplet. Thus a logarithmic growth law for the time dependent length scale $L(t)$ of the droplet excitations is assumed to hold. Instead of the thermal relaxation time t for a classical process, we use in our quantum case the quantum tunnelling rate Γ_L for a droplet of linear size L. So the length scale $L(t)$ growth is determined not by the thermal activation over the free energy barriers between minima but by quantum fluctuations which cause a droplet tunnelling through the barrier at rates that do not vanish for $T \to 0$. The fraction of droplets which are quantum-mechanically active at $T \to 0$ is proportional to Γ_L.

In general, quantum effects could change nonequilibrium dynamics in glassy phase at very low temperatures.

The paper is organized as follows. In the next section we give the definitions and the main properties of the spin glass droplet model. In Section 3 we give the linear response formalism and the general expression for the *ac* magnetic susceptibility. In Section 4 we present a summary of our results and conclusions.

2 The model and Hamiltonian

In this paper we use a phenomenological quantum droplet model within the spin glass theory [36–38] (which does not use the mean-field approximation) in order to describe the nonequilibrium behavior of the magnetic dynamical susceptibility at very low (but finite) temperatures T. We use the quantum Hamiltonian for the short-range Ising spin glass in a transverse field. This model Hamiltonian may be appropriate for experimental systems such as the dipolar magnet $LiHo_xY_{1-x}F_4$, proton glasses, alkali halides with tunneling impurities and other quantum systems [9,36–38].

We shall be interested in the behavior at very low temperatures in the ordered spin glass phase and ignore critical effects.

The droplet model describing the low-dimensional short-range Ising spin glass is based on renormalization group arguments [35,36]. In dimensions above the lower critical dimension d_l (usually in spin glass $2 \le d_l < 3$) it gives a low temperature spin-glass phase in zero magnetic field. This phase differs significantly from the spin-glass phase in the mean-field approximation of the Sherrington-Kirkpatrick infinite-range spin-glass model [9]. In the droplet model there are only two pure thermodynamical states related to each other by a global spin flip. In the presence of a magnetic field there is no phase transition. A droplet is an excited compact cluster in an ordered state where all the spins are inverted. The natural scaling ansatz for the droplet free energy ϵ_L (which is considered to be independent random variable) is $\epsilon_L \sim L^{\theta}$, with $L \ge \zeta(T)$; ζ is the correlation length, L is the length scale of droplet and θ is the zero temperature thermal exponent. One droplet consists of a number of spins of order L^d. Below d_l, $\theta < 0$; above d_l one has $\theta > 0$. The droplet excitations have a broad distribution of their free energies with the probability distribution $P_L(\epsilon_L)d\epsilon_L$ of droplet free energies at scale L for large L in a scaling form [4,6]

$$P_L(\epsilon_L)d\epsilon_L = \frac{d\epsilon_L}{\gamma(T)L^\theta}\mathcal{P}\left(\frac{\epsilon_L}{\gamma(T)L^\theta}\right),\ L \to \infty. \quad (5)$$

It is assumed that $P_L(x \to 0) > 0$, $\mathcal{P}_L(0) - \mathcal{P}_L(x) \sim x^\phi$ at $x \to 0$; $\phi \leq 1$. $\gamma(T)$ is the stiffness constant of the domain wall on the boundary of the droplet which is of order of the characteristic exchange $\mathcal{I} = \overline{(\mathcal{I}_{ij}^2)}^{\frac{1}{2}}$ at $T = 0$ and vanishes for $T \geq T_g$. At positive temperatures the droplet free energy will replace the droplet energy [35] but for brevity we shall simply write droplet energy.

The Hamiltonian of the d-dimensional quantum Ising spin glass in a transverse field is given by

$$\mathcal{H} = -\sum_{i,j}\mathcal{I}_{ij}S_i^zS_j^z - \Gamma\sum_i S_i^x \quad (6)$$

where S_i are the Pauli matrices for a spin at site i. Γ is the strength of the transverse field and the sum in (6) is performed over nearest neighbors. For $\Gamma = 0$ the Hamiltonian (6) describes the d-dimensional classical Ising spin glass. The interactions $\mathcal{I}_{ij}$ are independent random variables of mean zero and variance $\mathcal{I} = \overline{(\mathcal{I}_{ij}^2)}^{\frac{1}{2}}$. This model describe, for example, the physics of the proton glasses, the mixed betaine phosphate-phosphite [9]. The transverse field may be interpreted as the frequency of the proton tunnelling. Finally, an experimental realization of the quantum Ising spin glass is the dilute dipole coupled magnet $\mathrm{Li\,Ho}_x\mathrm{Y}_{1-x}\mathrm{F}_4$ where quantum fluctuations are introduced and controlled by means of a transverse magnetic field [9]. The properties of model (6) in mean-field approximation have been studied in many papers [6,9]. It was found that there is a high-temperature critical behavior at temperature $T_c(\Gamma) \sim \mathcal{I}$, $\Gamma \ll \mathcal{I}$ and a low-temperature critical behavior with the zero-temperature critical point $T_c(\mathcal{I}) = 0$, $\Gamma_c(0) = \mathcal{I}$, where Γ_c is the critical value of Γ below which the spin-glass phase can exist. We suppose as in our early papers [37,38] that a quantum system with the Hamiltonian (6) has a true glass phase transition at $T_g \neq 0$.

One can use the Suzuki-Trotter formalism [36] to show that the d-dimensional quantum mechanical system (6) is equivalent to a classical statistical mechanical system in $(d+1)$-dimensions (the extra dimension is the imaginary time) with the classical Hamiltonian

$$\mathcal{H}_{cl} = \Delta\tau\sum_{k=1}^{L_\tau}\sum_{\langle ij\rangle}\mathcal{I}_{ij}S_{i,j}S_{j,k} - \mathcal{I}_F\sum_{k=1}^{L_\tau}\sum_i S_{i,k}S_{i,k+1}. \quad (7)$$

Here the variables $S_{i,k}(=\pm 1)$ denote classical Ising spins, representing the z-component of the quantum spins at site i and imaginary time $\tau = k\Delta\tau$ (the imaginary time direction has been divided into L_τ time slices of width $\Delta\tau$). The calculation gives $\exp(\mathcal{I}_F) = \tanh(\Delta\tau\Gamma)$, with $\Delta\tau L_\tau = (k_BT)^{-1}$ (here and throughout the paper we use units where $\hbar = 1$). Referring to the Hamiltonian (7) at temperature $T = 0$, Thill and Huse [36] have assumed that in each of the two ordered states (for $\Gamma < \Gamma_c$ and $T < T_g$) for sufficiently low T and an appropriate range of length scales L, the quantum Hamiltonian (6) can be represented as low energy droplets (analogous to independent quantum two-level systems in a structural glass) with the Hamiltonian

$$\mathcal{H} = \frac{1}{2}\tilde{\sum_L}\sum_{D_L}\left(\epsilon_L S_{D_L}^z + \Gamma_L S_{D_L}^x\right) \quad (8)$$

where $S_{D_L}^z$ and $S_{D_L}^x$ are the Pauli matrices representing the two states of the droplet. The sum is over all droplets D_L at length scale L and over all length scales L, and

$$\tilde{\sum_L} \sim \int_{L_0}^\infty \frac{dL}{L} \quad (9)$$

where L_0 is a short-distance cutoff. ϵ_L is the droplet energy which is the independent random variable. The droplet length scale L is greater than or of the order of the correlation length ζ. The quantity

$$\Gamma_L = \Gamma_0\exp[-\sigma L^d] \quad (10)$$

is the tunnelling rate for a droplet of linear size L, Γ_0 being the microscopic tunnelling rate; σ is the surface tension for the interface between the two droplet states, which is approximately the same for all droplets. We will assume that Γ_L is the same for all droplets of scale L [36]. The Hamiltonian of a single droplet is the 2×2 matrix

$$\frac{1}{2}\begin{pmatrix}\epsilon_L & \Gamma_L\\ \Gamma_L & -\epsilon_L\end{pmatrix} \quad (11)$$

with eigenvalues $E_\pm = \pm\sqrt{\epsilon_L^2 + \Gamma_L^2}$. $E = 2|E_\pm|$ is the energy difference between the two eigenvalues.

In the quantum droplet model of Thill and Huse [36] the relative reduction of the Edwards-Anderson order parameter $q_{EA}(T)$ from its zero-T value $q_{EA}(0)$ is given for $\theta > 0$ and $L^*(T) \geq \zeta$ by

$$1 - \frac{q_{EA}(T)}{q_{EA}(T=0)} \sim \frac{k_BT}{\gamma L^{*\theta}(T)} = \frac{k_BT}{\gamma}\frac{\sigma^{\theta/d}}{\ln^{\theta/d}(\Gamma_0/(k_BT))} \quad (T \to 0) \quad (12)$$

where the crossover length scale is

$$L^*(T) = \left(\frac{1}{\sigma}\ln\frac{\Gamma_0}{k_BT}\right)^{\frac{1}{d}}. \quad (13)$$

For droplets with $L \ll L^*(T)$ and $\Gamma_L \gg k_BT$ the excitation energy $\sqrt{\epsilon_L^2 + \Gamma_L^2}$ is always greater than k_BT and thermal fluctuations are therefore not essential at temperature T. These droplets behave quantum-mechanically while larger droplets ($L \gg L^*(T)$) have $\Gamma_L \ll k_BT$ and behave classically. These large droplets ($\epsilon_L \leq k_B T$, $\Gamma_L \leq k_B T$) are thermally active and reduce q_{EA}. For $\theta > 0$ this reduction is dominated by the smallest droplets. Below d_l ($\theta < 0$) the thermally-excited droplets disorder the system reducing q_{EA} to zero at any temperature.

In this paper we neglect droplet-droplet and droplet-lattice interactions.

3 Linear response

We consider the time-dependent Hamiltonian $\hat{\mathcal{H}}$ of the quantum system in the form [46]

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}'(t) = \hat{\mathcal{H}}_0 - \hat{A}h(t) \tag{14}$$

where $\hat{\mathcal{H}}_0$ is an unperturbed part describing the equilibrium system. We suppose that the external perturbation $\hat{\mathcal{H}}'(t)$ is in some sense small, with $\hat{A}$ being the linear operator through which the external time varying force $h(t)$ couples to the system.

We evaluate quantum-mechanically the dynamical response $\Delta\hat{B}(t) \equiv \langle\hat{B}(t)\rangle - \langle\hat{B}\rangle_0$ to the force $h(t)$ in terms of the time-evolution operator $\hat{U}(t,t')$; here $\hat{B}(t)$ is the Heisenberg operator, $\hat{B}(t) = \hat{U}^\dagger(t,t')\hat{B}(t_0)\hat{U}(t,t')$, while $\langle\hat{B}\rangle_0$ is the equilibrium expectation value of $\hat{B}$. Performing a standard first-order perturbation expansion of $\hat{U}(t,t')$ one gets

$$\hat{U}(t,t') \simeq \hat{U}_0(t,t')\left\{\hat{1} - \frac{i}{\hbar}\int_{t'}^{t} dt_1 \hat{U}_0^\dagger(t_1,t') \times \hat{\mathcal{H}}'(t_1)\hat{U}_0(t_1,t')\right\} \tag{15}$$

where $\hat{U}_0(t,t') = \exp\left[-\frac{i}{\hbar}(t-t')\hat{\mathcal{H}}_0\right]$ and the sign $\dagger$ means the conjugate value.

We consider a response functional of the form [46]

$$\Phi(t,t';t_0) \equiv \frac{1}{i\hbar}\langle[\hat{A}(t_0),\hat{B}(t,t')]\rangle_0 \tag{16}$$

where $\langle\ldots\rangle_0$ denotes the thermal average with the density matrix $\hat{\rho}_0 \equiv \hat{\rho}(t_0)$, t_0 being the time when the perturbating field is turned on.

Now we apply the aforementioned dynamical response relations to a magnetic system. Then the response $\langle\hat{B}(t)\rangle$ represents the induced magnetization $M(t)$ and $\langle\hat{B}\rangle_0$ is the equilibrium magnetization M_0. Let a small magnetic oscillating field

$$h(t) = h\exp[i\omega t] \tag{17}$$

be applied in z-direction where h and ω are the amplitude and the angular frequency of *ac* field. Then we look for the induced magnetization in the z-direction.

In order to observe a history dependence and aging in a spin glass, the sample is quenched infinitely fast at zero *dc* magnetic field from a temperature $T \gg T_g$ to the temperature $T_1 < T_g$ which is reached at the time $t = 0$. At this moment a very small external magnetic oscillating field $h(t)$ is applied to measure the *ac* susceptibility of the sample. The evolution continues in isothermal conditions and χ_{ac} is measured at fixed frequency ω as a function of the time t elapsed since the sample reached the temperature T_1.

The system is probed at a time t after the quench end (the "age"). Using linear response theory the magnetization of the magnetic system is [25]

$$M(t) - M_0 = \int_0^t \chi(t,t_1)h(t_1)dt_1 = \int_0^t \chi(t,t-t')h(t-t')dt' \tag{18}$$

where $\chi(t,t-t')$ is the dynamical magnetic susceptibility determining the magnetic response at time t to a unit magnetic field impulse at time $(t-t')$. The nonequilibrium processes are probed by the low-frequency *ac* susceptibility measurements. The frequency dependent *ac* susceptibility is measured by applying a *ac* magnetic field $h(t)$ at time $t = 0$. Then $\chi(\omega,t)$ may be found by the Fourier transform of the magnetization over a time segment t_m ($t_m \sim 2\pi/\omega$) centered around t [25,41]

$$\chi(\omega,t) = \frac{1}{t_m}\int_{t-\frac{t_m}{2}}^{t+\frac{t_m}{2}} dt'' e^{-i\omega t''} \times \int_0^{t''} dt' \chi(t'',t''-t')e^{i\omega(t''-t')}. \tag{19}$$

If the magnetic response function varies little over the time segment t_m the susceptibility $\chi(\omega,t)$ will be equal to [25]

$$\chi(\omega,t) = \int_0^t dt' \chi(t,t-t')e^{-i\omega t'}. \tag{20}$$

The in-phase component of the *ac* susceptibility is $\chi'(\omega,t) = \mathrm{Re}\chi(\omega t)$, while the out-of-phase component is $\chi''(\omega,t) = \mathrm{Im}\chi(\omega t)$.

We consider the behavior of the magnetic droplet system described by the Hamiltonian (8) under an applied *ac* field $h(t)$ in the quantum regime ($\Gamma_L \gg k_B T$) when the droplet excitation energy $\sqrt{\epsilon_L^2 + \Gamma_L^2}$ is greater than $k_B T$. In the calculations presented below, we assume that $\theta > 0$ ($d > d_l$). There is a complicated classical-to-quantum crossover depending on the temperature T, the frequency of the *ac* field ω and the length scale L. According to [36], the dynamical crossover length is determined from the condition $\Gamma_L^{-1} = t$, i.e.

$$L^*_{dyn}(T) \sim \left(\frac{\sigma}{\Delta}k_B T\right)^{\frac{1}{\psi - d}}. \tag{21}$$

The system behaves presumably classically or quantum-mechanically when the dominant length scale L is above or below L^*_{dyn} for fixed frequency ω. When the droplets behave quantum-mechanically they have a characteristic rate, the Rabi frequency, which is of order Γ_L.

Following the aforementioned quantum droplet theory with the model Hamiltonian (8) and relating it to domain growth ideas, we calculate the *ac* susceptibility using the dynamical response functional including the first- and second-order linear response functions [46]. The contribution of a single droplet to the *ac* susceptibility up to some factor $\sim q_{EA}^2 L^{2d}$ is

$$\chi_{D_L}(\omega,t)-\chi_{D_L}(\omega=0)\sim -\tanh(\beta a_L/2)\sin^2\varphi\frac{a_L}{a_L^2-\omega^2}$$
$$-h\tanh(\beta a_L/2)\cos\varphi\sin^2\varphi\Bigg(\Bigg\{\frac{(7a_L^2-10\omega^2)\cos(\omega t)}{(a_L^2-\omega^2)(a_L^2-4\omega^2)}$$
$$-\frac{3a_L\sin(\omega t)\sin(a_L t)+6\omega\cos(\omega t)\cos(a_L t)}{\omega(a_L^2-4\omega^2)}-\frac{\cos(a_L t)}{a_L^2-\omega^2}\Bigg\}$$
$$+i\Bigg\{\frac{3(a_L^2+2\omega^2)\sin(\omega t)}{(a_L^2-\omega^2)(a_L^2-4\omega^2)}$$
$$-\frac{3a_L\cos(\omega t)\sin(a_L t)-6\omega\sin(\omega t)\cos(a_L t)}{\omega(a_L^2-4\omega^2)}$$
$$+\frac{3\sin(a_L t)}{\omega(a_L^2-\omega^2)}\Bigg\}\Bigg), \qquad (22)$$

where $a_L=\sqrt{\epsilon_L^2+\Gamma_L^2}$, $\sin\varphi=\Gamma_L/a_L$, $\cos\varphi=\epsilon_L/a_L$ and $\chi_{D_L}(\omega=0)$ is the static susceptibility of the droplet D_L. The expression (22) was obtained for low frequencies satisfying the condition $\omega t\geq 1$ (and $\Gamma_L<\omega$) because this condition is used to observe nonstationary dynamics in χ_{ac} measurements [12]. Now we have to average the susceptibility (22) over droplet energies ϵ_L and over droplet length scales L. In order to average over droplet energies we use the distribution $P_L(\epsilon_L)$ (5). In this distribution we assume $\phi=0$. We approximate in (22) $\tanh(\beta\sqrt{\epsilon_L^2+\Gamma_L^2}/2)\simeq 1$ and integrate as shown in references [36–38]. Upon averaging over the droplet energy, the contribution of all droplets of the system to the real part of susceptibility is obtained in the following form

$$\chi_L'(\omega,t)-\chi_L'(\omega=0)\sim\frac{\Gamma_L^2}{\gamma L^\theta\omega\sqrt{\omega^2-\Gamma_L^2}}$$
$$\times\left(\ln\frac{\omega+\sqrt{\omega^2-\Gamma_L^2}}{\omega-\sqrt{\omega^2-\Gamma_L^2}}-\ln\frac{\omega p_L+\sqrt{\omega^2-\Gamma_L^2}b_L}{\omega p_L-\sqrt{\omega^2-\Gamma_L^2}b_L}\right)$$
$$+\frac{\Gamma_L^2 h}{\gamma L^\theta\omega^3}\Bigg[\sin(\omega t)\Bigg(\frac{3}{2}\mathrm{si}(b_L t)+\mathrm{si}(t(b_L-\omega))+\mathrm{si}(t(b_L+\omega))$$
$$+\frac{3}{4}\mathrm{si}(t(b_L-2\omega))+\frac{3}{4}\mathrm{si}(t(b_L+2\omega))\Bigg)$$
$$+\cos(\omega t)\Bigg(-\frac{3}{4}\mathrm{ci}(t(b_L-2\omega))+\frac{3}{4}\mathrm{ci}(t(b_L+2\omega))-\ln\frac{b_L+\omega}{b_L-\omega}$$
$$-\frac{3}{4}\ln\frac{b_L+2\omega}{b_L-2\omega}-\mathrm{ci}(t(b_L-\omega))+\mathrm{ci}(t(b_L+\omega))+\frac{5\omega}{b_L}$$
$$-\frac{3\omega}{b_L}\cos(b_L t)-3\omega t\,\mathrm{si}(b_L t)\Bigg)-\frac{2\omega}{b_L}\cos(b_L t)-2\omega t\,\mathrm{si}(b_L t)\Bigg]. \qquad (23)$$

Here $\chi_L'(\omega=0)$ is the static susceptibility of the system of droplets of size L, $b_L=\sqrt{p_L^2+\Gamma_L^2}$, $p_L=2\omega+\Gamma_L$, $\mathrm{si}(\alpha)$ is the sine integral and $\mathrm{ci}(\alpha)$ is the cosine integral. Further we average expression (23) over length scales L. While integrating over L we see that the real part of the susceptibility is dominated by droplets of length scale $L_1\sim[(1/\sigma)\ln(\Gamma_0/\omega)]^{1/d}$. L_1 is the natural length scale of the problem when $\Gamma_L\sim\omega$ and represents the lower limit of integration over L. The upper limit of integration was taken as $L_2\sim[(1/\sigma)\ln(t_0\Gamma_0)]^{1/d}$. The average over L finally leads to the following expression for the real part of the *ac* susceptibility of the droplet system

$$\chi'(\omega,t)-\chi'(\omega=0)\sim$$
$$\frac{\Gamma_0^2}{\gamma\omega^2}\Bigg(\left(\ln\frac{4\omega^2}{\Gamma_0^2}-2\right)(2\sigma)^{\frac{\theta}{d}}d^{-1}\mathrm{G}\left[-\frac{\theta}{d},2\left|\ln\frac{\Gamma_0}{\omega}\right|\right]$$
$$+\frac{\Gamma_0^2}{4\omega^2}\left(2\ln\frac{4\omega^2}{\Gamma_0^2}-1\right)(4\sigma)^{\frac{\theta}{d}}d^{-1}\mathrm{G}\left[-\frac{\theta}{d},4\left|\ln\frac{\Gamma_0}{\omega}\right|\right]$$
$$+\frac{3\Gamma_0^4}{8\omega^4}(6\sigma)^{\frac{\theta}{d}}d^{-1}\mathrm{G}\left[-\frac{\theta}{d},6\left|\ln\frac{\Gamma_0}{\omega}\right|\right]$$
$$+\sigma\Bigg(2(2\sigma)^{\frac{\theta}{d}-1}d^{-1}\mathrm{G}\left[1-\frac{\theta}{d},2\left|\ln\frac{\Gamma_0}{\omega}\right|\right]$$
$$+\frac{\Gamma_0^2}{\omega^2}(4\sigma)^{\frac{\theta}{d}-1}d^{-1}\mathrm{G}\left[1-\frac{\theta}{d},4\left|\ln\frac{\Gamma_0}{\omega}\right|\right]\Bigg)\Bigg)$$
$$+\frac{\Gamma_0^2 h}{\gamma\omega^3}\Bigg(\Bigg(\frac{1}{\omega t}\Bigg(\frac{3}{16}\sin(3\omega t)-\frac{17}{12}\sin(2\omega t)$$
$$-\frac{3}{4}\sin(\omega t)\cos(2\omega t)\Bigg)+\cos(\omega t)\left(\frac{5}{2}-\ln 3-\frac{3}{4}\ln\frac{4\omega}{\Gamma_0}\right)\Bigg)$$
$$\times\Bigg(-d^{-1}(2\sigma)^{\frac{\theta}{d}}\left(\mathrm{G}\left[-\frac{\theta}{d},2\left|\ln(t\Gamma_0)\right|\right]-\mathrm{G}\left[-\frac{\theta}{d},2\left|\ln\frac{\Gamma_0}{\omega}\right|\right]\right)\Bigg)$$
$$-\frac{3}{4}\sigma\cos(\omega t)\Bigg(-d^{-1}(2\sigma)^{\frac{\theta}{d}-1}\Bigg(\mathrm{G}\left[1-\frac{\theta}{d},2\left|\ln(t\Gamma_0)\right|\right]$$
$$\times-\mathrm{G}\left[1-\frac{\theta}{d},2\left|\ln\frac{\Gamma_0}{\omega}\right|\right]\Bigg)\Bigg)\Bigg) \qquad (24)$$

where $\mathrm{G}[\alpha,x]$ is the incomplete gamma function. This is the main result of our paper.

If we use the asymptotic representation for the incomplete gamma function for large values of the its second argument we obtain, for example, for the difference of two incomplete gamma functions

$$\mathrm{G}\left[1-\frac{\theta}{d},2\left|\ln(t\Gamma_0)\right|\right]-\mathrm{G}\left[1-\frac{\theta}{d},2\left|\ln\omega^{-1}\Gamma_0\right|\right]$$
$$\sim|\ln(\omega^{-1}\Gamma_0)|^{-\frac{\theta}{d}}\left[1-\frac{1}{(\omega t)^2}\left(\frac{|\ln(\omega^{-1}\Gamma_0)|}{\ln(t\Gamma_0)}\right)^{\frac{\theta}{d}}\right]\frac{\omega^2}{\Gamma_0^2}. \qquad (25)$$

We observe some similarity with expression (5.7) in reference [35] in conformity with our quantum regime.

In the derivation of equation (24) we made the following approximations: (1) $\mathrm{si}(b_L t)\simeq-(p_L t)^{-1}\cos(p_L t)$; (2) $\mathrm{ci}(b_L t)\simeq(p_L t)^{-1}\sin(p_L t)$; (3) $b_L\simeq 2\omega+\Gamma_L$.

We also evaluated numerically the average over L of the expression (23) without these approximations and obtained curves for the susceptibility which are very similar as those obtained from (24) (the parameter values are given below).

The susceptibility $\chi'(\omega, t)$ depends on the external *ac* magnetic field and on several parameters of the droplet system. Among them, we mention the kind of distribution function $P_L(\epsilon_L)$, and the droplet microscopic tunnelling rate Γ_0.

The expression (24) consists of time-independent terms which describe the simple oscillations with frequency ω, and terms which depend on time t and determine the nonstationary nonequilibrium dynamics of the droplet system. So, the real part of the *ac* susceptibility can be represented approximately as a sum of the stationary part (χ'_{ST}) and nonstationary part (χ'_{NST})

$$\chi'(\omega, t) \simeq \chi'_{ST} + \chi'_{NST}\,. \tag{26}$$

This separation is found to be exact in mean field models.

For a numerical evaluation of the expression (24) we take the following values of the parameters: $d = 3$, $\theta = 0.5$, $\gamma = 10^{-15}$, $\Gamma_0 = 10^8$, 10^{10}, 10^{12}, $h = 1$, $\sigma = 10^{-15}$, $t = 0 \div 100$, $\omega = 0.05$, 0.1.

In Figure 1 we show the t-dependence of the real part χ'_{NST} of the *ac* susceptibility of the droplet system. Here ωt is comparable or more than unity, so one may observe nonstationary dynamics and the aging regime [12]. In Figure 1a we show the slow dynamics at $\Gamma_0 = 10^8$, $\omega = 0.05$, 0.1 and $t = 0 \div 25$. At short elapsed times t the curve grows (very quickly) up to some finite value and then falls down. At longer times t the *ac* susceptibility shows a stationary behavior. In Figure 1b and Figure 1c the t-dependence of $\chi'_{NST}(\omega, t)$ is shown for longer times: (b) $t = 0 \div 50$; (c) $t = 0 \div 100$. The time interval covers two decades of the elapsed time t. We see that as frequency is increased the susceptibility magnitude decreases and the slow dynamics is suppressed at higher frequency. In Figures 1 we see the influence of the frequency ω and the elapsed time t on the susceptibility.

In Figure 2 we give the time dependence of $\chi'_{NST}(\omega, t)$ at different values of the quantum parameter Γ_0 ($= 10^8$, 10^{10}, 10^{12}) giving the microscopic tunnelling rate. We observe a small effect of Γ_0 on the susceptibility $\chi'_{NST}(\omega, t)$. With increasing values of Γ_0 the magnitude of the susceptibility slightly decreases at small times t, i.e. the quantum fluctuations in some sense lead to a decrease of the susceptibility in quantum regime in the spin glass phase.

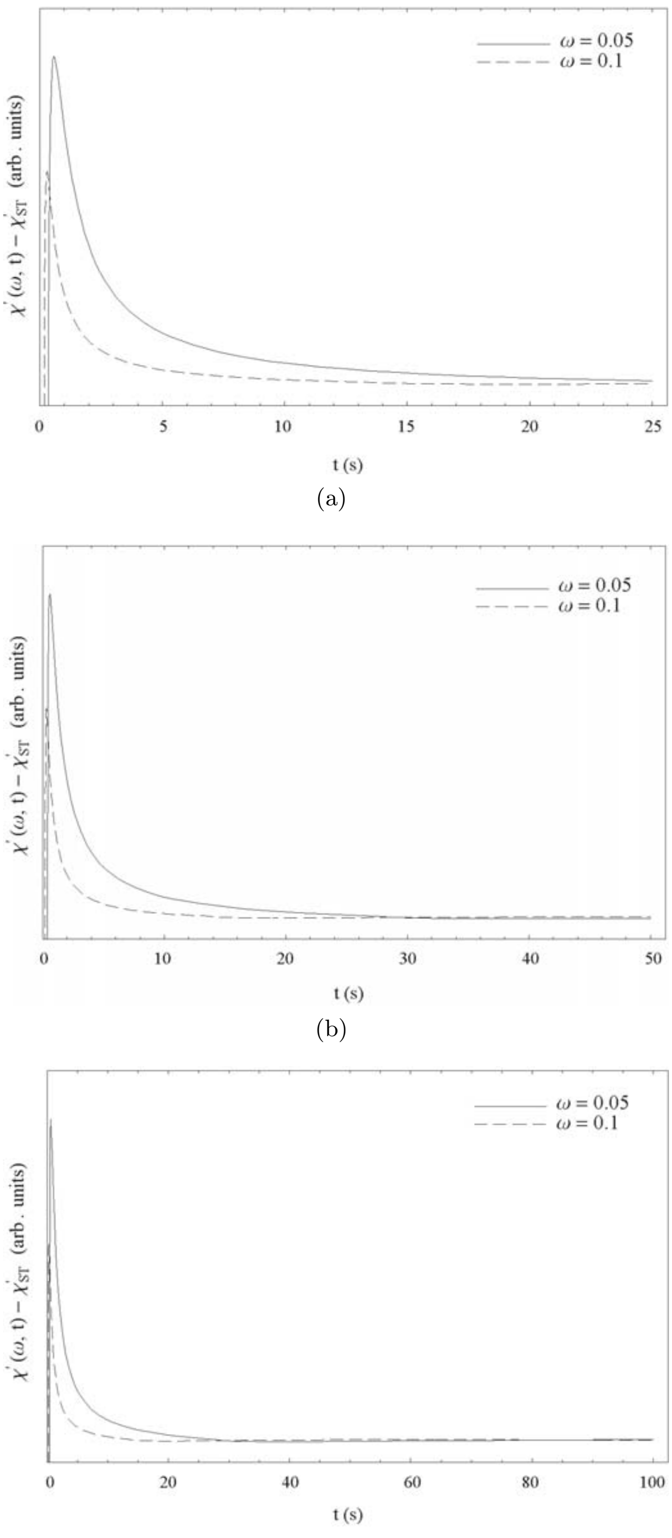

Fig. 1. The in-phase susceptibility $\chi'(\omega, t)$ (nonstationary part) as a function of time t for quantum parameter $\Gamma_0 = 10^8$ and fixed values of frequency $\omega = 0.05, 0.1$: (a) $\omega = 0.05, 0.1$; $t = 0 \div 25$; (b) $\omega = 0.05, 0.1$; $t = 0 \div 50$; (c) $\omega = 0.05, 0.1$; $t = 0 \div 100$.

4 Discussion and conclusion

The main aim of this paper is to show the role of quantum fluctuations in the behavior of the *ac* susceptibility of our quantum system. In the paper we have investigated the low temperature nonequilibrium dynamical behavior of the magnetic *ac* susceptibility in d-dimensional Ising

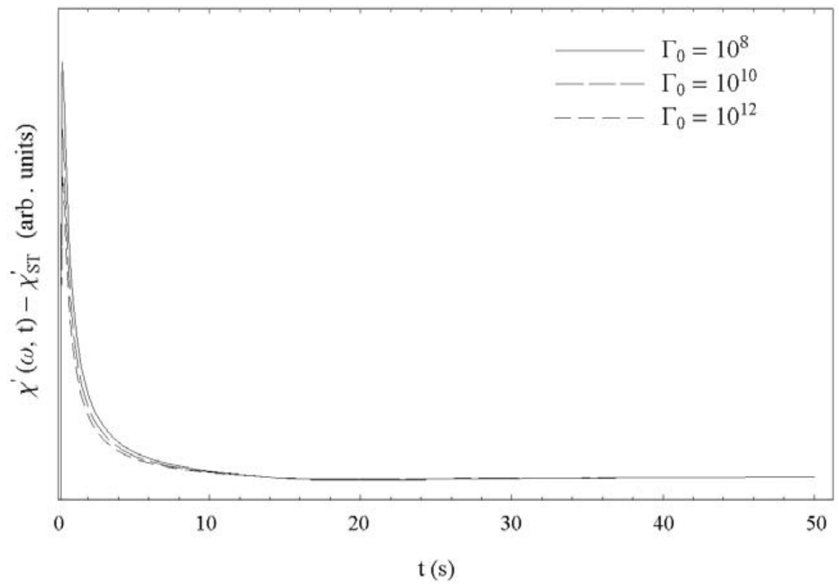

Fig. 2. The in-phase susceptibility $\chi'(\omega, t)$ (nonstationary part) as a function of time t for frequency $\omega = 0.1$ and for three values of quantum parameter $\Gamma_0 = 10^8$, $\Gamma_0 = 10^{10}$, $\Gamma_0 = 10^{12}$.

spin glass with short-range interactions between spins in a transverse field in terms of the phenomenological droplet model. The real part of the low-frequency *ac* susceptibility $\chi'(\omega, t)$ as a function of the time t elapsed from the initial quench up to the the measurement and of the frequency ω of the external *ac* magnetic field is calculated. We display the nonequilibrium dynamics for different low values of ω at constant temperature in the spin glass phase. The real part of the *ac* magnetic susceptibility $\chi'(\omega, t)$ of the droplet system at very low temperatures (quantum regime) has two time regions in which the time evolution is of a different nature. At short elapsed times ($t < t_{char}$) we observe a nonequilibrium dynamical behavior, manifest in the fast decay of $\chi'(\omega, t)$ at low frequency and at constant temperature T. t_{char} is some characteristic time which defines the two regimes with stationary dynamics of $\chi'(\omega, t)$ and with nonstationary one. At longer times t ($t > t_{char}$) the curve becomes a simple periodic function oscillating around some constant value (stationary process).

We have shown that the quantum fluctuations have slight influence on the dynamical susceptibility of the droplet system at very low temperatures. Their increase leads to a small decrease of the susceptibility magnitude. If the frequency of the *ac* field increases the nonequilibrium dynamics is suppressed. So, the response of the droplet system to an external perturbing field weakly depends on the thermal history.

In [40] it was shown that the behavior of the response function $R(t, t_w)$ demonstrates the existence of the stationary and aging regimes in quantum systems. The theoretical curve (Fig. 2 in [40]) $R(t, t_w)$ was given as function of τ ($\tau = t - t_w$), with $\tau \in [0, 50]$ and $t_w = 2.5$, 5, 10, 20 and 40 (t_w is the waiting time). For $\tau \leq \tau_{char}$ (τ_{char} is some characteristic time) a stationary regime was found, whereas for $\tau > \tau_{char}$ the dynamics is nonstationary. In reference [40] it is shown that quantum fluctuations in quantum glassy systems depress the phase transition temperature, in a glassy phase aging survives the quantum fluctuations and the quantum fluctuation-dissipation theorem is modified due to quantum fluctuations. In reference [39] it is shown that in the aging regime of quantum spin glasses of rotors all terms in the dynamical equations governing the time evolution of the spin response and the correlation function that arise solely from quantum effects are irrelevant at long times. The quantum effects enter only through the renormalization of the dynamical equations parameters [43]. In reference [47] the nonequilibrium dynamics of a quantum Heisenberg spin glass with a nontrivial $SU(N)$ spin algebra is considered. In this paper the numerical evidence for aging and for a generalized fluctuation-dissipation theorem in the aging regime is given. The behavior of the spin response as a function of τ ($\tau = t - t_w$) is similar to the behavior of the dynamic susceptibility in our paper. In reference [47] it is shown that the aging regime is not affected by quantum fluctuations and the quantum system behaves classically in its slow evolution.

As far as we know there are no experiments on quantum spin glasses. In references [18,24,25] experimental data on $\chi'(\omega, t)$ in classical spin glasses are presented. Among them, Svedlindh et al. [25] have investigated the behavior of $\chi'(\omega, t)$ and $\chi''(\omega, t)$ and have found that decay is close to a logarithmic one, Shins et al. [24] have observed that $\chi'(\omega, t)$ decreases with time in a nearly logarithmic way, whereas in reference [18] it was found for $\chi'(\omega, t)$ a behavior which resembles our curves in Figures 1. So, in the aging regime a slow monotonous decay of $\chi'(\omega, t)$ was observed. In our quantum system at very low T we cannot find agreement with these data because classical and quantum spin glasses have in general different behavior.

Thus, we have found the nonequilibrium low-frequency dynamical behavior of the *ac* susceptibility at constant temperature for the droplet system in quantum regime ($\Gamma_L \gg k_B T$) in the spin glass phase. For small times t the magnetic *ac* susceptibility depends both on the frequency of the *ac* magnetic field ω and on the time t elapsed since the sample reached the temperature T_1 ($T_1 < T_g$). We do not find a slow continue decrease of the amplitude of $\chi'(\omega, t)$ as a function of t at long times.

We may compare our data with experimental ones in classical spin glasses [18,24,25] only very approximately because in the experiments on $\chi'(\omega, t)$ and in our paper qualitatively different spin-glass systems are considered. We observe a qualitatively similar dynamical behavior of $\chi'(\omega, t)$ in the range of the small times elapsed since the quench.

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References

1. P.G. de Gennes, Rev. Mod. Phys. **71**, 5374 (1999)
2. E.R. Nowak, J.B. Knight, E. Ben-Naim, H.M. Jaeger, S.R. Nagel, Phys. Rev. E **57**, 1971 (1998)
3. M. Nicokas, P. Duru, O. Pouliquen, Eur. Phys. J. E **3**, 309 (2000)
4. H.M. Jaeger, C.-H. Liu, S.R. Nagel, Phys. Rev. Lett. **62**, 40 (1989)
5. E. Falcon, R. Wunenburger, P. Evereque, S. Fauve, C. Chabot, V. Garrabos, D. Beysens, Phys. Rev. Lett. **83**, 440 (1999)
6. *Spin-glasses and random fields,* edited by A.P. Young (World Scientific, Singapore, 1998)
7. L.C.E. Struik, *Physical aging in amorphous polimers and other materials* (Elsevier, Houston, 1978)
8. D. Fisher, P. Le Doussal, C. Monthus, Phys. Rev. E **64**, 066107 (2001)
9. J.A. Mydosh, *Spin Glasses: an experimental introduction* (Taylor & Francis, London, 1993); J. Brooke, D. Bitko, T.F. Rosenbaum, G. Aeppli, Science **284**, 779 (1999)
10. R.L. Leheny, S.R. Nagel, Phys. Rev. B **57**, 5154 (1998)
11. F. Alberci-Kious, J.-Ph. Bouchaud, L.F. Cugliandolo, P. Doussineau, A. Levelut, Phys. Rev. Lett. **81**, 4987 (1998)
12. E. Vincent, J. Hammann, M. Ocio, J.-Ph. Bouchaud, L.F. Cugliandolo, in *Complex behavior of glassy systems* edited by M. Rubi (Springer Verlag, Berlin, 1997)
13. J.-Ph. Bouchaud, L.F. Cugliandolo, J. Kurchan, M. Mezard, in *Spin Glasses and Random Fields* edited by A.P. Young (World Scientific, Singapore, 1998)
14. P. Nordblad, P. Svedlindh, in *Spin Glasses and Random Fields*, edited by A.P. Young (World Scientific, Singapore, 1998)
15. D.A. Huse, Phys. Rev. B **43**, 8673 (1991)
16. L. Berthier, J.-Ph. Bouchaud, Phys. Rev. B **66**, 054404 (2002)
17. P.E. Jonsson, H. Yoshino, P. Nordblad, H. Agura Katori, A. Ito, Phys. Rev. Lett. **88**, 257204 (2002)
18. V. Dupuis, E. Vincent, J.-Ph. Bouchaud, J. Hammann, A. Ito, H. Agura Katori, Phys. Rev. B **64**, 174204 (2001)
19. Y.G. Joh, R. Orbach, G.G. Wood, J. Hammann, E. Vincent, J. Phys. Soc. Jpn **69**, 215 (2000); Suppl. A
20. E. Vincent, V. Dupuis, M. Alba, J. Hammann, J.-Ph. Bouchaud, Europhys. Lett. **50**, 674 (2000)
21. V.S. Zotev, G.F. Rodriguez, G.G. Kenning, R. Orbach, E. Vincent, J. Hammann, Phys. Rev. B **67**, 184422 (2003)
22. C. Djurberg, J. Mattsson, P. Nordblad, Europhys. Lett. **29**, 163 (1995)
23. L. Lundgren, P. Nordblad, P. Svedlindh, O. Beckman, J. Appl. Phys. **57**, 3371 (1985)
24. A.G. Shins, A.F. Arts, H.W. de Wijn, Phys. Rev. Lett. **70**, 2340 (1993)
25. P. Svedlindh, K. Gunnarsson, J.-O. Andersson, Phys. Rev. B **46**, 13867 (1992)
26. P. Shukla, S. Singh, Phys. Rev. B **23**, 4661 (1981)
27. T.K. Kopec, Phys. Rev. B **54**, 3367 (1996)
28. T. Giamarchi, P.L. Doussal, Phys. Rev. B **53**, 15206 (1996)
29. T. Nieuwenhuizen, Phys. Rev. Lett. **74**, 4289 (1995)
30. J. Miller, D.A. Huse, Phys. Rev. Lett. **70**, 3147 (1993)
31. D. Grempel, M. Rozenberg, Phys. Rev. Lett. **80**, 389 (1998)
32. H. Rieger, A.P. Young, *Quantum spin-glasses* (Springer-Verlag, Berlin, 1996)
33. J. Schwinger, J. Math. Phys. **2**, 407 (1961); L.V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964)
34. C. Chamon, A.W.W. Ludwig, C. Nayak, Phys. Rev. B **60**, 2239 (1999); A. Kamenev, A. Andreev, Phys. Rev. B **60**, 2218 (1999); M.N. Kiselev, R. Oppermann, Phys. Rev. Lett. **85**, 5631 (2000)
35. D.S. Fisher, D.A. Huse, Phys. Rev. Lett. **56**, 1601 (1986); D.S. Fisher, D.A. Huse, Phys. Rev. B **36**, 8937 (1987); D.S. Fisher, D.A. Huse, Phys. Rev. B **38**, 373, 386 (1988)
36. M.J. Thill, D.A. Huse, Physica A **241**, 321 (1995)
37. G. Busiello, R.V. Saburova, Int. J. Mod. Phys. B **14**, 1843 (2000)
38. G. Busiello, R.V. Saburova, V.G. Sushkova, Solid. State Commun. **119**, 545 (2001); G. Busiello, R.V. Saburova, V.G. Sushkova, J. Phys. Studies **5**, 1 (2001)
39. M.P. Kennett, C. Chamon, J. Ye, Phys. Rev. B **64**, 224408 (2001)
40. L.F. Cugliandolo, G. Lozano, Phys. Rev. Lett. **80**, 4979 (1998); L.F. Cugliandolo, G. Lozano, Phys. Rev. B **59**, 915 (1999)
41. T. Komori, H. Yoshino, H. Takayama, J. Phys. Soc. Jpn **68**, 3387 (1999); T. Komori, H. Yoshino, H. Takayama, J. Phys. Soc. Jpn **69**, 1192 (2000)
42. H. Yoshino, K. Hukushima, H. Takayama, Phys. Rev. B **66**, 064431 (2002)
43. H.E. Castillo, C. Chamon, L.F. Cugliandolo, M.P. Kennett, Phys. Rev. Lett. **88**, 237201 (2002)
44. V.S. Zotev. G.G. Kenning, R. Orbach, Phys. Rev. B **66**, 014412 (2002)
45. J. Kisker, L. Santen, M. Schreckenberg, Phys. Rev. B **53**, 64181 (1996)
46. W.T. Grandy Jr, *Foundation of statistical mechanics* (D. Reidel Publishing Company, Dordrecht, Holland, 1988); H.J. Kreuzer, *Nonequilibrium thermodynamics and its statistical foundations* (Charendon Press, Oxford, 1981)
47. G. Biroli, O. Parcollet, Phys. Rev. B **65**, 094414 (2002)